

A Rough Guide to the KnotPlot Demos

Rob Scharein

August 11, 2022

KnotPlot comes with a wide variety of demos that you can access using the **DemoA** and **DemoB** tabs on the KnotPlot Control Panel. Some of these demos are “serious” and intended to illustrate important concepts in Knot Theory. Others are less so and are designed to be playful or decorative. A few of the demos are fairly deep exploratory tools, that can be quite engaging for the entrepid knot enthusiast. Others are simply a waste of time. Most demos are randomized to some extent and this means you will see something different everytime you run them. Some demos have a large amount of random variation, some have little (or even none). Click and find out!

This document briefly explains the point, if any, of most of the demos. It is organized by sections, starting with the **Knot theory** section on **DemoA**.

Knot theory

guess knot — Try to guess which of the two objects you see is really knotted. One of the objects can be smoothly deformed into a circle without having the string pass through itself (so it’s an *unknot*). The other cannot (being a *non-trivial knot* called a *trefoil*). This is an illustration of the most important problem in knot theory: when is something really knotted? This is known as the *unknotting problem* and is actually a difficult problem.

Things to try: (1) use the **compare to...** buttons to compare the two objects before clicking on **find out!**. (2) After running the demo once, try changing the point of view (left click and drag in the main view window and rotate the knot) and then reload the knot (or unknot). ¹

(un)knot ? — Similar in function to **guess knot** but whether or not this beast is really knotted might be more obvious.

Perko pair — A famous pair of knots that were listed as distinct knots in knot tables until Ken Perko showed in 1974 [Per74] that they were in fact the same knot. His proof consisted of a sequence of knot diagrams. This is a different proof by KnotPlot. The knots start out in different conformations and end up looking exactly the same. The *knot equivalence problem* turns out to be even more difficult than the unknotting problem previously mentioned.

See knotplot.com/perko

Brunnian — Starts off with five round circles and a string interwoven through them that comes from page 67 of Rolfsen [Rol76]. Click on **run animation** to find out what happens. This is a link that has the *Brunnian Property* (after the mathematician Brunn). The individual strings in a link with this property cannot be separated. However, *if any one string is removed*, the others fall apart. Follow the on-screen instructions

¹This complicated unknot is sometimes called *The Monster* and may have been created by Ken Millett.

and see if this is true.

See youtube.com/watch?v=vshcgnSUtyI, knotplot.com/brunnian and knotplot.com/brunnian/borromean-rings.html

random braid — Related to knot theory is *braid theory*. Braids are special kinds of knot diagrams, of the type you see when you click on the **random braid** button. Any knot can be constructed by closing some braid. Section 3.4 of [Sch98] for more information.

Redm. moves — The three fundamental moves in diagrammatic knot theory. In a sequence of knot diagrams drawn on a piece of paper, two knots are equivalent if and only if you can convert one diagram into the other via a sequence of Reidemeister Moves.²

chirality — Some knots are the same as their mirror image. Most are not. This demo has a proof that one of the knots is equivalent to its mirror image. Such a knot is called *achiral*. The other knot may or may not be equivalent to its mirror image.³ Most knots are not achiral and cannot be smoothly deformed into their mirror image. These are known as *chiral* knots.

twist knot — Shows how a particular family of knots, the *twist knots*, are constructed.

torus knot — Another family of knots, the *torus knots*, are those knots that can be drawn (or wrapped around) a torus. Use the KnotPlot **torus** command or the **Cons** control panel to create any torus knot or link.

For a pretty picture, visit knotplot.com/knot-theory/torus_xing.html

satellite knot — Shows the construction of a *satellite knot* inside a trefoil. Arbitrary satellite knots are easily created using KnotPlot's **braid** and **warp** commands. See Section 5.4.2 of my thesis [Sch98] for details.

Lissajous — Yet another infinite family of knots, the *Lissajous knots* [BHJS94]. These knots have a fairly simple formula. Use the **lissajous** command or the **Cons** control panel to create your own Lissajous knots.

tangles — Embed little tangles into a *tangle holder* to create complex knots and links. The tangle holders are mostly due to John H. Conway [Con70] (he called them *basic polyhedra*).

rat tang — Illustrates how a class of tangles, the *rational tangles*, are created. Arbitrary rational tangles can be created with the **tangle** command. For example, try one of the following:

tangle (3/2)

tangle (11/7)

ACN — Visualization of how the *Average Crossing Number (ACN)* is computed. There are actually two demos here, use the button at the bottom of the view window to toggle between them. The first demo is static whilst the other is dynamic (try tugging the knot around, instructions are given).

BB frame — Constructing the *blackboard framing* of a knot. Doesn't always work.

moebius — Performs *Möbius inversion* through a 2-sphere.

magnetic — Shows what part of the magnetic field lines around a knot would look like if the knot were a current carrying wire.

Things to try: (1) Click on **two rings** or **hopf link** and then hover your mouse over one of the strings until you see some text appear. Then click your mouse to reverse the current in that string. Observe the change in the magnetic field. (2) If things drift apart, click on **collide**.

²Along with smooth deformations of the paper.

³The other knot is the trefoil and it's not equivalent to it's mirror image, but this is not so easy to show.

codes — Interactive computation of DT and Gauss codes. These are topological encodings of a knot diagram useful in knot classification studies. You can do a lot of things with this demo, be sure to click on the ? button in the KnotPlot View Window.⁴

Constructions & Relaxations

Split/delete — An illustration of the power of KnotPlot’s knot simplification techniques. This uses two opposing processes: getting rid of extraneous string in loose areas and adding extra string in tight spots. All this whilst preventing the knot from passing through itself. This method works well for all the examples seen in the **Knot zoo**: section on the left of the KnotPlot View Window, with the exception of one. Can you find it? KnotPlot has trouble with this knot, but a hint is given on how to untie the nasty unknot with a little help from the user (you). For more information on this topic, see Section 7.3.3 of my thesis [Sch98].

Forces — Have fun playing with the different force laws that KnotPlot uses to relax knots. Full control of these forces can be found on the Dyna control panel (or using the command window).

Free Drag — Pull a string around in 3D. Try to make a knot!

weave — A weaving machine. Click the **show / hide weaver** button to reveal the weaver. Any knot can be used as a weaver, this demo uses only torus knots.

ringpack — Pack rings until you fill up some space. You can change the ring size and thickness and you can pack things other than rings. Ring packing of this sort is related to percolation theory and also has important applications in studying how DNA molecules are arranged in certain organisms. Demo inspired by a talk by Yuanan Diao (click the ? button for more info and a citation).

hedgehog, crankshaft — An illustration of the *hedgehog algorithm* and the *crankshaft algorithm*, two methods commonly used to create random knots. Click the ? button for instructions, information and citations (crankshaft algorithm is due to Ken Millett).

chains, rchains — Create knotted chains.

cable, braid warp — Watch a cable knot or a warped braid get constructed. Keep clicking on **rerun demo**, some of them are fun to watch as they are constructed. Notice that you can rotate the knot as it’s being constructed.

shadows — The “shadow” of a knot. The shadow appears to deform, but the knot itself is rigid. It is simply undergoing a smooth rotation and we are viewing its shadow onto a plane.

gravity — Uses the gravity force in KnotPlot. Has three separate sub-demos, the funnest of which is **drop**. Try clicking on this repeatedly for an hour or more in fullscreen mode.

Lorenz — Unfinished demo of knotted paths along the *Lorenz Attractor*.

THK — Create a family of knots known by the politically incorrect name “Turk’s Head Knots”. These are constructed from torus knots by forcing the knot to be alternating.

Hopf — Visualizes the *Hopf fibration*. This demos is quite engaging in fullscreen mode. Click on one of the **seed** buttons. Each button produces something different every time you click on it. When in fullscreen

⁴It’s a good idea to click on the ? button any time when you see it in the KnotPlot View Window. It’s always in the lower right and usually brings up some useful information.

mode, press the spacebar to hide or show the text and buttons.

plaits, splaits — Uses the Celtic Knot Construction Engine (CKCE) inside KnotPlot to create complex plaits, somewhat in the Celtic style. “Splaits” are spherical plaits.⁵

star/box — Also uses the CKCE to create star like or boxes with a Celtic flavour.

ant neck — First few steps along the way to the construction of *Antoine’s Necklace*, a fractal knot. Note that Antoine constructed this fractal in his imagination and clearly understood its properties nearly half a century before Benoit Mandelbrot claimed to have discovered the concept of a fractal.

trapped — Trap a knot inside a cage constructed using the CKCE.

simple — An assortment of knots undergoing dynamical relaxations. Sometimes the knot explodes. There is little to control here, but some of the knots can be fun to watch.

Thing toss — Toss either rings, strings or things through a loop in the centre of the window. After you’ve tossed a few, click on the follow button to ride along with the tossed object.

Anchors — A poor or at best incomplete illustration of the use of KnotPlot anchors.

Graphs — KnotPlot is not limited to plotting knots. It can also plot graphs, and these can be knotted. The knot theory of graphs is an extension of standard knot theory. It is a relatively unexplored area that has important applications in biology and chemistry.

Thing toss — on a Mac, try turning the sound on (turn down the volume first).

NeverEnding exhibitions

The demos in this section are of a different nature than all the other demos on either DemoA or DemoB in that when engaged, the demos will take over the full screen and run without interrupting until one presses the ESC key.

Knots! — The most complete of the eight exhibits. A mixture of serious knot theory and KnotPlot fun. About a dozen or so different sequences. The sequences repeat, but in an unpredictable order.

Fly thru — Fly along knotted roller coasters. Every few seconds, the path changes.

Frenetic — Just what it says.

Poly dance — Similar to Fly thru but only the outlines of the polygons are shown.

Paint 1, Paint2 — Doesn’t work properly on all graphics hardware. On some computers, it will appear to flicker. This is not intentional.

Aum Fly — Aums floating in space.

glue graphs — Knotted graphs with attitude. Watch little circles try to escape the chaotic jumble only to get pulled into the tangled mess. Why are these things so agitated?

⁵To create your own Celtic Knots, click on the Sketch button on the KnotPlot Control Panel and then click on → Celtic knotting near the bottom of that control panel.

References

- [BHJS94] M. G. V. Bogle, J. E. Hearst, V. F. R. Jones, and L. Stoilov. Lissajous knots. *Journal of Knot Theory and Its Ramifications*, 3(2):121–140, 1994.
- [Con70] J. H. Conway. An enumeration of knots and links, and some of their algebraic properties. In John Leech, editor, *Computational Problems in Abstract Algebra*, pages 329–358, 1970.
- [Per74] Kenneth A. Perko. On the classifications of knots. *Proceedings of the American Mathematical Society*, 45:262–266, 1974.
- [Rol76] Dale Rolfsen. *Knots and Links*. Publish or Perish, Inc., 1976.
- [Sch98] Robert G. Scharein. *Interactive Topological Drawing*. PhD thesis, Department of Computer Science, The University of British Columbia, 1998.